COMPOUND SYSTEMS USING SCALAR THEORY OF DIFFRACTION: AN ENGINEERING TOOL FOR OPTICAL DESIGN AND OPTOMETRY

ABOUT THE AUTHOR

Habib Hamam

Affiliate
Université de Moncton, Nouveau-Brunswick, Canada

Research field
Physics with focus on Telecommunications,Electrical and Communications Engineer, Engineering

Email
habib.hamam@cit.edu.al

Abstract: Geometrical optics offers the formalism of compound systems by using for instance the Gauss' method. This formalism notably simplifies the analysis of image formation. Since this theory does not take into account the wave character of light, it is desirable to extend the formalism of compound systems to scalar theory of diffraction which offers a more accurate tool of analysis. This paper treats this extension and shows its importance. For illustration, the system of human eye is considered. The analysis is simplified by using a lenses-based model of the Fresnel transform.

Key words: geometrical optics, scalar theory of diffraction, Fresnel transform, compound systems.

1. Introduction

The purpose of this paper is not to develop a geometrical theory of diffraction as done by Keller and others [1-3] but to define a useful tool in analogy with the compound system in geometrical theory. We will start at the beginning from scalar theory of diffraction and aim at defining such a tool for easily analyzing and designing optical systems in the framework of scalar theory of diffraction. The optical behavior of systems including several diffractive optical elements is more visible and easier to analyze if all these elements are combined into one compound system. As a consequence, the synthesis of diffractive systems satisfying predefined constraints becomes easier if such a compound system exists in the framework of scalar theory of diffraction. Moreover, this compound system can be extended to optical applications involving phenomena analogous to diffraction in mathematical terms. A typical application is wave propagation inside a single-mode fiber that is approximated by a dispersive medium of second order [4-5]. The compound system would be very useful, for example, in the synthesis of fiber components compensating chromatic dispersion. These components are key elements in high bit rate optical telecommunications (>10 Gbit/s).

The elements of originality may be summarized as follows: The formalism of geometrical optics notably simplifies the analysis of image formation. However, this formalism is not rigorous and is based on a rough approximation, namely considering light propagating in straight lines (rays). The present work proposes a more rigorous formalism as well as an engineering tool offering the same simplicity of the analysis of image formation as in geometrical optics. It also applies on the system of human eye. Thus, optometrists and ophthalmologist could use the proposed engineering tool. This tool uses a lenses-based model.

After a brief overview on geometrical optics and on diffraction, we treat in the analysis section the Fresnel transform that is a powerful tool to model diffraction in the framework of scalar theory of diffraction. In the same section, we briefly cover the formalism of the compound system in the framework of the approximation of geometrical optics. Then we extend this formalism to scalar theory of diffraction and define the scalar-theory-based compound system. In the next section we consider the example of the spherical diopter as one compound system. For illustration, we consider another example, namely the optical system of the human eye. To simplify the analysis of diffraction, we advance a lenses-based model of the Fresnel transform.

1.1. Geometrical optics

Geometrical optics refers to the simple ray tracing techniques that have been used for centuries [5,6]. Its basic postulates include the following: (1) The wave direction is specified by the normal to the equiphase planes ("rays") (2) Rays travel in straight lines in a homogeneous medium (3) Power in a bundle of rays is conserved. (4) Reflection and refraction obey Snell-Descarte's law.

Given an object and an optical instrument, geometric optics cannot offer a full interpretation of the formation of its image in an arbitrary location. In addition to the geometrical aberrations, this technique faces a limit when the phenomenon of diffraction occurs. In this paper, we have the intention to overcome these limitations while profiting from the formalism of geometrical optics based compound system.

1.2. Diffraction

Grimaldi carried out a simple, but fundamental, experiment in which he illuminated an aperture in an opaque screen with a light source and observed the intensity across a plane at some distance behind the screen [8]. Grimaldi observed that the transition from light to shadow is gradual rather than abrupt whereas according to the corpuscular theory, the shadow behind the screen should be well defined with sharp borders. We should admit that the source used was mediocre and thus hindered Grimaldi from observing the presence of light and dark fringes in the geometrical shadow. This leads us to excuse the geometro-opticiens for not discovering diffraction sooner.

Despite Newton's support of the corpuscular theory [9] (He believed that the light propagation is a movement of corpuscles that respects the rules of mechanics and notably that of the universal gravitation), Huygens advanced the ondulatory theory (wave theory) based on Grimaldi's observations. He explained Grimaldi's observation by a purely intuitive postulation, in which he regarded light propagation as an incessant creation of elementary spherical light sources [10].

Like Huygens, Young, who discovered interference [11], supported the ondulatory theory. His belief in the analogy between light and sound leads him to state that light vibration is longitudinal [12]. The famous A. Fresnel was of the same opinion. However he considered that Huygens' postulation did not explain the non-existence of waves, that have the same specifications, propagating backwards. He combined Huygens' principle of the "envelope" building, with the interference principle of Young and, for the purpose of putting forward a coherent theory, he made some supplementary hypotheses on the amplitude and phase of the new elementary waves. At the end of the XIXth century, G. Kirchhoff gave a deeper mathematical basis to the diffraction theory introduced by Huygens and Fresnel, and considered Fresnel's hypothesis as a logical consequence of the ondulatory nature of light. Kirchhoff's work was subjected a few years later to criticisms made by Sommerfeld who considered the Kirchhoff formulation as a first approximation. He advanced with Rayleigh what was later called the "Rayleigh-Sommerfeld diffraction theory".

2. Analysis

The Rayleigh - Sommerfeld diffraction theory treats the propagation of light as a scalar phenomenon and thus neglect its electromagnetic nature: the electromagnetic field must be characterized by its two components, the electric and the magnetic field, which are coupled by the Maxwell's equations. In the scalar approach we consider only one transverse component of the field. This approximation is valid, however, if the diffracting object is large compared with the wavelength of light, if the observation distance is sufficiently large, and if the angles involved are small enough to guarantee that the axial field components can be neglected. Widely used approaches include the so-called Fresnel and Fraunhofer approximations [13], which describe the diffraction patterns in the Fresnel region and in the far field of the aperture, respectively.

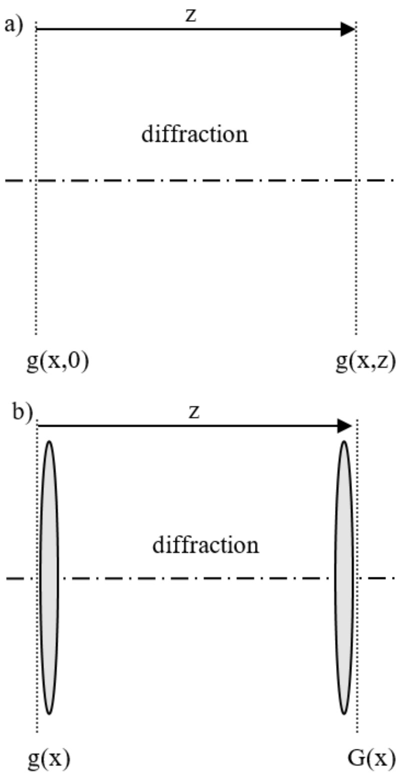

Figure 1: Fresnel and Fourier transforms: a) The diffraction field observed in the Fresnel zone at a distance *z* is expressed by the Fresnel transform *g(x,z)*. b) The Fourier Transform *G(x)* of a object *g(x)=g(x,0)* can be implemented by two identical convergent lenses separated by the focal length.

2.1. Fresnel transform

In this work, we opt for the Fresnel approximation given that we are interested in relatively far finite distances with respect to the object features. Moreover for brevity of notation, the analysis is limited to the one-dimensional consideration. Hence, the diffracted field *g(x,z)* observed at a distance *z* (fig. 1a) is expressed by the Fresnel transform [13], as follows:

$$g(x,z) = FR_z\{g(x,0)\} = \frac{\exp(i2\pi z/\lambda)\exp(-i\pi/4)}{\sqrt{\lambda z}}\, g(x) * f_k(x,z) \qquad (1)$$

where *g(x)=g(x,0)* is the initial field, λ is the wavelength in the wavelength in vacuum and * denotes convolution. The initial field is also referred to as the object (a complex object). It might be an aperture function, a diffractive mask, an analog or digital hologram, etc. The Fresnel kernel is expressed as follows:

$$f_k(x,z) = \exp\left(i\pi\frac{x^2}{\lambda z}\right) \qquad (2)$$

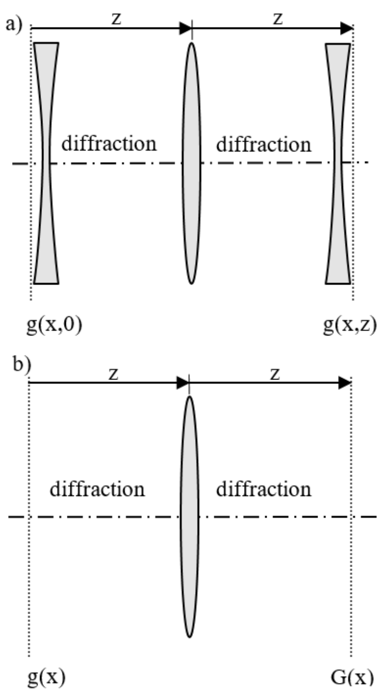

Figure 2: Another way to respectively implement the Fresnel and Fourier transforms of figure 1: a) using two divergent lenses of focal length −*z* and a convergent lens of focal length *z*. b) using one focal length of focal length *z*.

For a wave propagating in a medium with a refraction index n, the wavelength λ should be replaced by λ/n in equations (1) and (2). For brevity of notation, the constant term of propagation $\exp(i2\pi z/\lambda)$ and the factor $\exp(-i\pi/4)/\sqrt{\lambda z}$ will be ignored. If we move to the Fourier plane, equation (1) becomes [14]:

$$G(u,z) = G(u)\exp\left(-i\pi\lambda z u^2\right) \qquad (3)$$

where *G(u,z)* is the Fourier transform of the diffracted field *g(x,z)* and *G(u)* is the Fourier transform of the initial field : *g(x,0)=g(x)*.

In addition to relation (1), the diffraction field observed at a distance *z* can be expressed by a Fourier Transform (FT):

$$g(x,z)=\exp\left(i\pi\frac{x^2}{\lambda z}\right)FT\left\{\exp\left(i\pi\frac{u^2}{\lambda z}\right)g(u)\right\}\Big|_{u'=x(\lambda z)} \quad (4)$$

This relation is known as the generalized diffraction equation or Collins equation [15]. After calculating the FT, the spatial frequency *u'* is replaced by *x/(λz)*. The two quadratic terms inside and outside the Fourier Transform argument represent two identical divergent spherical waves with a radius z. As a consequence of relation (4), to strictly obtain a FT of the initial field we should neutralize these two spherical waves by using two identical convergent lenses with a focal length *z* (fig. 1b). Moreover, we know that the FT can be implemented by a single spherical lens, where the object *g(x)* should be placed in its front focal plane [13]. The FT of this object is then observed in the back focal plane of the lens (fig. 2b). Thus, to obtain the Fresnel transform of *g(x)* we need two elements introducing the two quadratic terms of relation (4). This job can be done by two divergent spherical lenses (fig. 2a). This lenses-based model of the Fresnel transform, i.e of diffraction in the Fresnel regime, will be used to easily build compound systems using Gaussian formula analogously to geometrical optics.

2.2. Compound systems using geometrical optics

For simplicity of the study, we will analyze the concept of compound systems by means of a concrete simple example, namely the combination of two thin lenses in air. The generalization to more complex systems is straightforward.

Problems involving thin lenses in combination can be solved by successive application of the thin lens formulae. To calculate the position of the image formed by a system composed of two lenses, we can do it in two steps. We use the thin lens formula and calculate the position of the image formed by the first lens in isolation. We then consider this image as the object for the second lens to calculate the final image position.

An alternative to model image formation in the framework of geometrical optics approximation is to use the “paraxial ray propagator matrix” [16]. Two thin lenses in air are combined with a separation d as shown in Figure 3. Since the thickness of a thin lens is negligible, the lens matrixes of lens 1 and lens 2 are:

$$A_1=\begin{bmatrix}1 & 1/f_1\\ 0 & 1\end{bmatrix} \quad \text{and} \quad A_2=\begin{bmatrix}1 & 1/f_2\\ 0 & 1\end{bmatrix} \quad (5)$$

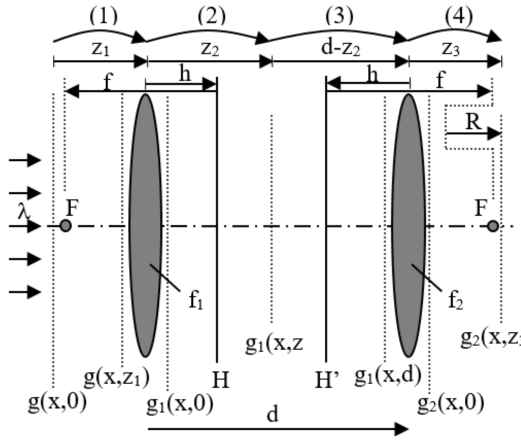

Figure 3: Optical setup including two lenses with focal lengths *f1* and *f2* separated by a distance d. Given *g(x,0)*, we can obtain the output field $g_2(x,z_3)$ by successively calculating of the diffractive field through the step (1) to (4).

The transfer matrix from lens 1 to lens 2 is:

$$T_2=\begin{bmatrix}1 & 0\\ -d & 1\end{bmatrix} \quad (6)$$

So the system matrix of the two-lens compound is:

$$A=A_2T_2A_1=\begin{bmatrix}A_1 & A_2\\ A_2 & A_2\end{bmatrix}=\begin{bmatrix}1-d/f_2 & 1/f_1+1/f_2-d(f_1.\\ -d & 1-d/f_1\end{bmatrix} \quad (7)$$

The anterior and posterior focal lengths *f* and *f'* (locations of F and F' in Figure 3) of the compound system are: (8)

$$\begin{cases}\dfrac{1}{f'}=A_{12}=\dfrac{1}{f_1}+\dfrac{1}{f_2}-\dfrac{d}{f_1f_2}\\ f=-f'\end{cases}$$

The location of the principal planes is determined by: (9)

$$h=\frac{1-A_{11}}{A_{12}}=d\frac{f'}{f_2} \quad \text{and} \quad h'=\frac{A_{22}-1}{A_{12}}=d\frac{f}{f_1}$$

The condition to obtain an image $g_2(x,z_3)$ of the object $g(x)=g(x,0)$ is indicated by the Gaussian formula :

$$z_3 = \frac{A_{21} - A_{22} z_1}{A_{11} - A_{12} z_1} \tag{10}$$

For simplicity of calculation, we will suppose that this condition is valid for the rest of the analysis. The linear magnification m is then:

$$m = \frac{A_{22} - z_3 A_{12}}{A_{11}A_{22} - A_{12}A_{21}} = \frac{1}{A_{11} - A_{12} z_1} = \frac{f_1 f_2}{f_1(f_2 - d) - z_1(f_2 + f_1 - d)} \tag{11}$$

The linear magnification or transverse magnification is the ratio of the image size to the object size.

2.3. Compound systems using scalar theory of diffraction

Let us continue with the two-lens compound system of Figure 3. The objective is to determine the output diffraction field $g_2(x,z_3)$ as a function of the input field $g(x)=g(x,0)$ and the system parameters, namely f1, f2 and d. To simplify the task, we track the diffraction field from the input to the output by dividing the diffraction process into four successive steps: (1) to (4) as indicated in the Figure 3. This leads to calculating the intermediate fields $g(x,z_1)$, $g_1(x,0)$, $g_1(x,z_2)$, $g_1(x,d)$ and $g_2(x,0)$. To avoid forbidding mathematical calculations by using the integral formulation of Fresnel diffraction (Eq. (1)), we prefer to use the lens based model of the Fresnel transform as illustrated in Figure 2a. The task will be very easy.

Fresnel diffraction through the steps (1) to (4) is then modeled by using the model of Figure 2a to finally obtain the setup of Figure 4. The two initial lenses of Figure 3 are colored in dark gray in Figure 4, whereas the lenses involved by the lenses-based model of the Fresnel transform are colored in light gray. The gray lenses will be called initial or original lenses. The diffraction through step (1) involves 2 divergent spherical lenses with focal length $-z_1$ and a convergent spherical lens with focal length z_1. At the end of the path (1), light comes cross the first initial lens with focal length f1 before covering the second path (2). This path also involves three lenses. The first divergent lens is located just behind the first original lens. Thus we obtain three lenses placed side by side as indicated in the left hand side of Figure 4. The optical behavior of these three lenses vanishes if the power of the positive lens is equal to the absolute value of the sum of the powers of the negative lenses:

$$\frac{1}{f_1} = \frac{1}{z_1} + \frac{1}{z_2} \tag{12}$$

Let us choose z_2 so that relation (12) is valid. These three lenses are then eliminated as indicated by a cross in Figure 4.

At the end of the path (2), we obtain two spherical lenses placed side by side (Figure 4). These two lenses can be replaced by a single lens with a power: $Fc = -1/z_2 + 1/(d-z_2)$, i.e. with a focal length:

$$f_c = \frac{1}{-1/z_2 - 1/(d - z_2)} = z_2 \frac{z_2 - d}{d} \tag{13}$$

Behind these two lenses, i.e. behind the compound lens with focal length fc, we obtain the field $g'(x)$ of Figure 4:

$$g'(x) = \exp\left(j\pi \frac{x_1^2}{\lambda z_1}\right) g(-x_1, 0) \exp\left(-j\pi \frac{x^2}{\lambda f_c}\right) \tag{14}$$

with $$x_1 = \frac{z_1}{z_2} x \tag{15}$$

The minus sign in the term g(-x1,0) comes from the succession of two Fourier transforms (telescope setup). The scale factor of relation (15) comes from the fact that the two Fourier transforms are undertaken with respect to two respective scale factors z_1 and z_2. For $z_1=z_2$, except for two quadratic phase terms (14) we obtain a telescope system without amplification: $x_1=x$ (4f-setup).

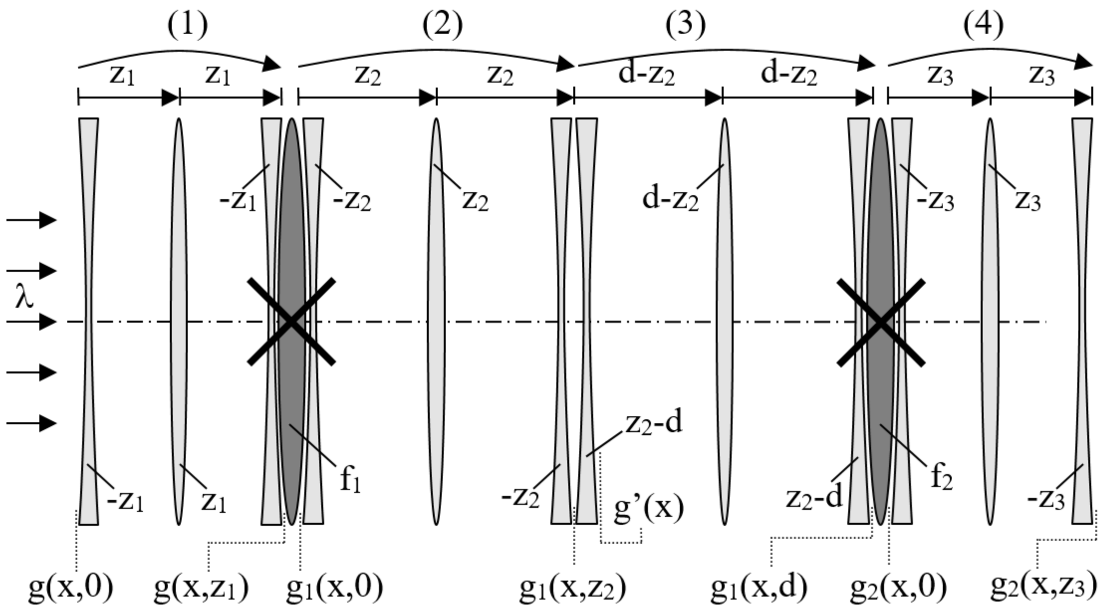

Figure 4: Modeling of the system of Figure 3 by the lenses based Fresnel transform model (fig. 2a). The two initial lenses of Figure 3 are colored in dark gray, whereas the lenses involved by the lenses based model of the Fresnel transform are colored in light gray.

Equation (14) combined with relation (15) gives:

$$g'(x) = \exp\left(j\frac{\pi}{\lambda}\left[\frac{z_1}{z_2^2} - \frac{1}{f_c}\right]x^2\right) g\left(-\frac{z_1}{z_2}x, 0\right) \quad (16)$$

Let us continue by considering the steps (3) and (4). Similarly to the paths (1) and (2) let us suppose that the three lenses placed side by side on the right hand side of Figure 4 satisfy the following condition:

$$\frac{1}{f_2} = \frac{1}{d - z_2} + \frac{1}{z_3} = \frac{z_3 + d - z_2}{z_3(d - z_2)} \quad (17)$$

We note that solving Eq (17) for z3 leads to Eq (9). We thus obtain the output field:

$$g_2(x, z_3) = g'(-x_2)\exp\left(j\pi\frac{x^2}{\lambda z_3}\right) \quad (18)$$

with $$x_2 = \frac{d - z_2}{z_3}x \quad (19)$$

Finally, we obtain:

$$g_2(x, z_3) = g'\left(-\frac{d - z_2}{z_3}x, 0\right)\exp\left(j\pi\frac{x^2}{\lambda z_3}\right) \quad (20)$$

or: $$g_2(x, z_3) = \exp\left(j\frac{\pi}{\lambda R}x^2\right) g\left(\frac{x}{S}\right) \quad (21)$$

where the radius of divergence R is defined as follows:

$$\frac{1}{R} = \left[\frac{z_1}{z_2^2} - \frac{1}{f_c}\right]\left(\frac{d - z_2}{z_3}\right)^2 + \frac{1}{z_3} \quad (22)$$

and the scaling factor S is:

$$S = \frac{z_2 z_3}{z_1(d - z_2)} \quad (23)$$

By combining relations (12), (17) and (23), we note that the scaling factor S is nothing but the linear magnification m of relation (11): $S=m$.
By combining relations (12), (17) and (22), we obtain the following expression of the radius R:

$$R = \frac{z_3(f_1 + f_2 - d) - f_2(f_1 - d)}{f_1 + f_2 - d} \quad (24)$$

Using relations (8), (9) and (24), we obtain after some algebra (Fig. 3):

$$R = z_3 - f' - h' \quad (25)$$

Thus, the quadratic term $\exp\left(j\frac{\pi}{\lambda R}x^2\right)$ of Eq.

(21) stands for a spherical wave starting from the posterior focal point F' of the compound system. Thus implies that if the input is a plane wave, the output wave is spherical and it

converges to the focal Point F'. Then, it continues as divergent spherical wave so that if we observe the field at a distance z_3, we then obtain the phase term: $\exp\left(j\frac{\pi}{\lambda R}x^2\right)$ In this case, the second term *g(x/S)* of Eq (21) is equal to 1 (input plane wave). This is in total agreement with geometrical optics.

Another interesting case is the situation of $z_3=h'$. We note that the distances of Figure 3 may be negative. The case $z_3=h'$ means that the observation plane is identical to the posterior principle plane H' of the compound system. It is expected, from geometrical optics, to obtain a spherical wave converging towards (or diverging from) the focal point F' if the input is a plane wave. According to Eq.(25), our special case means that $z_3=-f'$. Relation (22) implies that the observed field is $\exp\left(-j\frac{\pi}{\lambda f'}x^2\right)$, which corresponds to a spherical wave focusing on (or starting from) F'. This second special case is also in total agreement with geometrical optics.

2.4. A scalar theory based compound system

The previous analysis leads to the definition of a scalar theory system generalizing the geometrical optics based compound system. Figure 5 illustrates how the scalar theory based compound system works. Let us consider an input field *g(x,0)* propagating and coming across two lenses separated by a distance d. We first determine the focal length f of the compound system and the positions h and h' of the principal planes by using respectively relation (8) and (9). To obtain the expression of the output field *g'(x,z')*, the application of the scalar theory of diffraction consists in successively considering diffraction until the first lens (using relation (1)), the transmittance of the first lens, diffraction between both lenses, the transmittance of the second lens and finally diffraction behind the second system. To avoid this complexity, an equivalent system replaces the two separated lenses by a single one having the posterior focal length *f'*. In other words, the lenses are replaced by a system with a transmittance:

$$\exp\left(-j\pi\frac{x^2}{\lambda f'}\right)$$

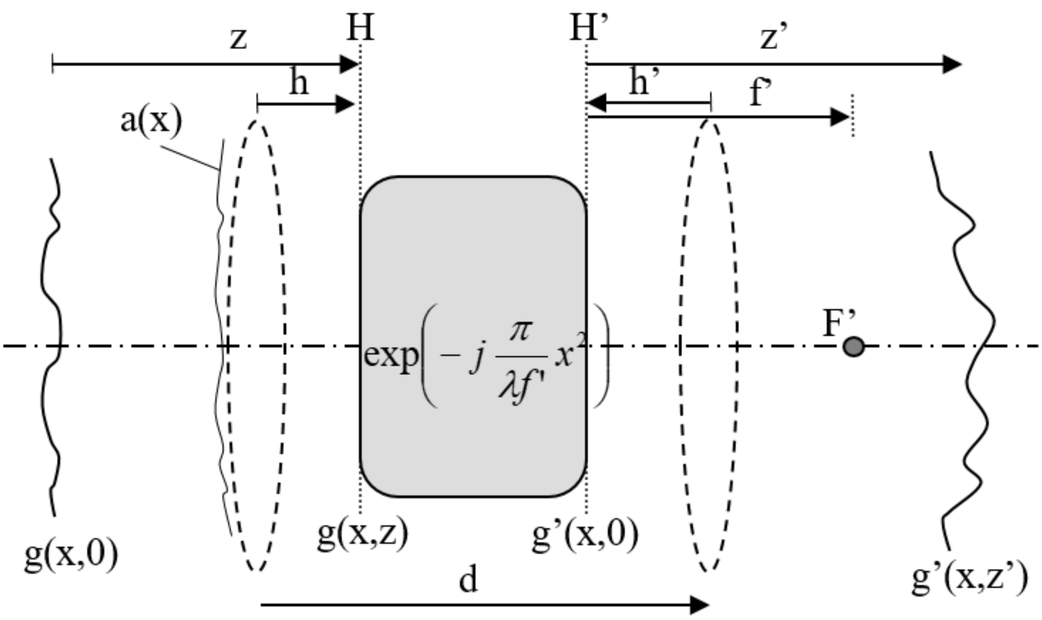

Figure 5: Scalar theory based compound system corresponding to the setup of Figure 3. The aberration field, *a(x)*, can be taken into account.

Thus we only need to calculate the diffraction field *g(x,z)* just before the compound system (using relation (1)). We then multiply the result by the transmittance of this system to obtain *g'(x,0)*. Finally, we reconsider diffraction along the distance *z'* to obtain the output field *g'(x,z')*. It is worth noting that the distance between the planes H and H' of Figure 5 becomes without any optical effect.

The inverse transform (starting from *g'(x,z')* to calculate *g(x)*) is also easy to undertake using the diffractive compound system. Therefore, the scalar theory based compound system is, in particular, very useful for the synthesis of diffractive elements satisfying constraints in the output plane. For example, using iterative methods such as the Gerchberg-Saxton algorithm [17,18], we iteratively modify the diffractive object *g(x,0)* so that its replay field, *g'(x,z')*, at the output of Figure 5 converges towards a form satisfying the constraints imposed by the application.

Aberrations can also be easily treated. If, for example, the first lens suffers from optical aberrations (Figure 5), the analysis remains valid and the wavefront aberration *w(x)* (expressed in meter or micron) can be taken into account. In this case, the diffraction field *g(x,z)* just before the compound system should be calculated in two steps (using relation (1) twice : two distances that are *z - h* then *h*) separated by a multiplication by *a(x)* (Figure 5). We note that the field *a(x)* caused by the presence of aberrations is linked to the wavefront aberration *w(x)* as follows:

$$a(x) = \exp\left(j\frac{2\pi}{\lambda} w(x) \right) \qquad (26)$$

For simplicity, we considered a system composed of two separated spherical lenses in the present analysis. Generalization to other systems is straightforward.

3. Example: spherical diopter

The spherical diopter can be modeled by the scalar theory based compound system. The principal planes H and H' of this system are superimposed and the nodal points are identical to the center of curvature C (Figure 6). Given that the field at the output propagates in a medium with a refraction index n_2, the transmittance of the system is then $\exp\left(-j\pi \frac{n_2 x^2}{\lambda f'} \right)$ where λ is the wavelength in vacuum.

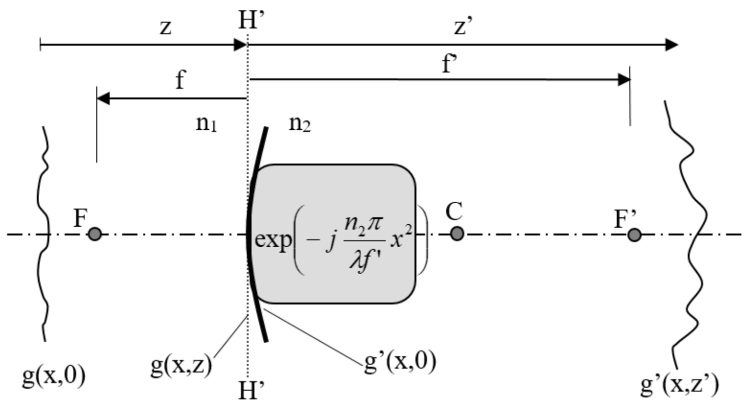

Figure 6: Spherical diopter as a compound system: at the surface separating the two media we apply a quadratic phase term.

For an input field *g(x,0)*, the diffraction field *g(x,z)* just before the diopter is expressed by relation (1) where the wavelength λ is replaced by λ/n1 (Figure 6). Then *g'(x,0)* is obtained by multiplying *g(x,z)* by the transmittance of the system, namely $\exp\left(-j\pi \frac{n_2 x^2}{\lambda f'} \right)$. Finally, *g'(x,z')* is obtained by applying relation (1) where the wavelength λ is replaced by λ/n2. For the special case where the incident wave is a spherical wave starting from the focal point F (Figure 6), according to geometrical optics, it is expected that the output wave be a plane wave (image at infinity). According to scalar theory, the diffraction field *g(x,z)* just before the spherical diopter is a spherical wave field expressed by the following quadratic phase term: $\exp\left(-j\pi\frac{n_1x^2}{\lambda f}\right)$ Just after the spherical surface separating the two media (n_1 and n_2), we obtain *g'(x,0)* merely by a multiplication of this phase term by the transmittance of the system yielding:

$g'(x,0)= \exp\left(-j\pi\frac{x^2}{\lambda}\left[\frac{n_1}{f}+\frac{n_2}{f'}\right]\right)$.Bearing in mind that the anterior and posterior focal lengths of a spherical diopter are linked as follows: $f'/n_2=-f/n_1$, we obtain a plane wave: *g'(x,0)= g'(x,z')=1* (the propagation constant term is neglected as mention in section 2.1). This is in full agreement with geometrical optics.

4. Application: the system of the eye

The scalar theory based compound system is useful for many fields of applications. For example, contrary to geometrical optics it allows analyzing the wavefront aberrations at any position in space. This analysis can be done for complex systems. Moreover, the analysis of the effect of diffraction, including the effect of the finite pupil size, is one of the main advantages scalar theory based models of optical systems. One of the system for which the scalar theory based compound system is very useful is the system of the eye (Figure 7). As an example, we consider one simplified eye model, namely the Gullstrand's simplified schematic eye [19-21]. Obviously, the reduced eye model or even other more complex models such as the schematic eye described by Emsley [22] can be also covered by the scalar theory based compound system.

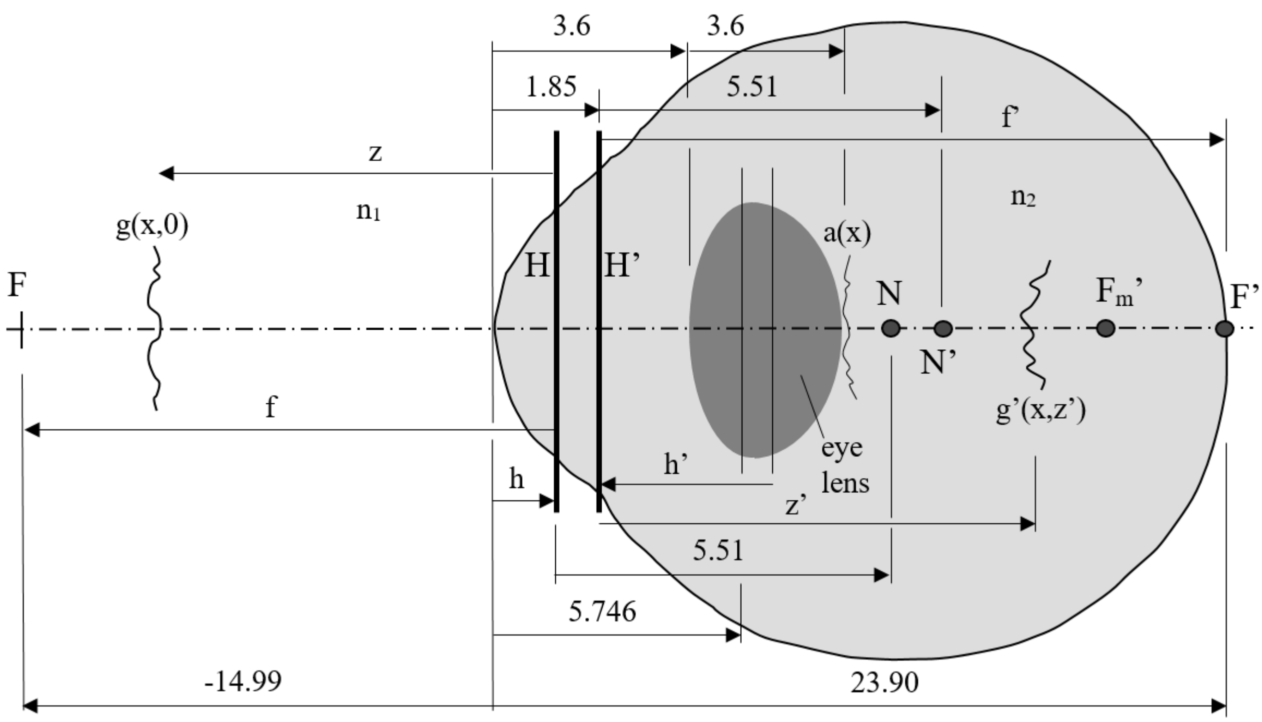

Figure 7: Gullstrand's simplified schematic eye: $f=-16.54$, $f'=22.05$, $h=1.55$, $h'=-4.06$, $n_1=1$ and $n_2=4/3$. N and N' are the nodal points. F' is the posterior focal point of an emmetropic eye, whereas F'_m corresponds to a myopic eye. Distances in the figure are in mm.

Given an input field *g(x,o),* the diffraction field *g(x,z)* observed in the principal plane H (Figure 7) is expressed by relation (1). Then, the field *g'(x,o)* in the plane H' is obtained by multiplying *g(x,z)* by the transmittance of the system,

namely $\exp\left(-j\pi\frac{n_2x^2}{\lambda f'}\right)$. The pupil function and aberrations can also be taken into account. The diffraction field *g'(x,z')* observed inside the eye at a distance *z'* is obtained by applying relation (1) where the wavelength λ is replaced by λ /n2. If, for example, one of the surfaces of the crystalline lens introduces aberrations (Figure 7), then the aberration field *a(x)* (relation (26)) should be considered in the calculation of *g'(x,z')*. This allows us to study the effect of aberrations associated to the individual optical components of the systems of the eye. We note that it is easier to take into account the aberration information concerning the outer components of the system (example: anterior cornea surface) than that of the inner components (example: anterior crystalline lens surface).

Figure 7 shows the case of an emmetropic eye (F' on the retina). The analysis is still valid if the eye is myopic (as pointed out in Figure 7, the focal point F'*m* is before the retina) or hyperopic. For an eye corrected with an ophthalmic or a contact lens or any other kind of correction, the scalar theory based compound system can be used. Two solutions are possible. First, we can calculate a compound system including the correction. The second solution consists in separately considering the correction element and the system of the eye yielding to an additional use of relation (1). This relation is then applied three times instead of twice. It is worth noting that aberrations associated to the correction (ophthalmic lens, intraocular lens, laser surgery, cornea implant, ...) can be easily integrated in the analysis.

5. Application: propagation in a dispersive optical fiber

The nonlinear Schrödinger equation governs the propagation of the optical pulses inside single-mode fibers [4]. For pulses larger than 1 ps, this equation is simplified as follows:

$$j\frac{\partial A}{\partial z}+\frac{j}{2}\alpha A-\frac{1}{2}\beta_2\frac{\partial^2 A}{\partial T^2}+\gamma|A|^2 A \qquad (27)$$

where A is the slowly varying complex amplitude of the pulse envelope, λ is the absorption coefficient, λ is the nonlinearity coefficient, λ 2 is the second order dispersion coefficient, z is the observation distance and the time T is measured in a frame of reference moving with the pulse at the group velocity vg *(T=t-z/vg)*. If neglect nonlinearity (λ =0) and normalize the complex amplitude and the time scale we obtain:

$$j\frac{\partial g}{\partial z}=\frac{1}{2}\beta_2\frac{\partial^2 g}{\partial T^2} \qquad (28)$$

where g(z, λ) is the normalized amplitude (Po : the peak power of the incident pulse):

$$A(z,\tau)=\sqrt{P_0}\exp(-\alpha z/2)g(z,\tau) \qquad (29)$$

and λ =T/To is the normalized time (To: pulse width). The Fourier transform of the solution of the differential equation (28) is:

$$G(\omega,z)=G(\omega)\exp\left(\frac{i}{2}\beta_2 z\omega^2\right) \qquad (30)$$

Equation (30) is similar to the diffraction equation (3) and becomes identical to it when λ 2 is replaced by - λ /2π . To compensate the dispersion effect cause by wave propagation inside the fiber, additional optical elements (Mach-Zender Modulators, chirped fiber gratings, ...) are introduced in the telecommunications chain. It is desirable that all these components are combined with the fiber itself in one compound system as we did with diffractive systems. Our analysis of diffraction systems in the previous sections can be straightforwardly extended to this application. Because the issue requires however some elaboration, it will be detailed in a future work.

6. Discussion

Given a combination of several optical components, it is useful to handle them as a single system having a determinable transmittance (or reflectance) function. Geometrical optics

offers this possibility by using for instance the Gauss' method. Since this theory does not take into account the wave character of light, scalar theory of diffraction offers a more accurate tool of analysis. For example, with the latter theory we can analyze higher diffraction orders.
We have shown that the definition of a compound system is possible in the framework of scalar theory. It allows to determine the output diffraction field at any distance z' as a function of the input field and its position in space z. The cost of this calculation is the application of the Fresnel transform twice and a multiplication by the transmittance (or reflectance) of the system. It also allows taking into account aberrations yielding however to additional calculations (generally an additional Fresnel transform). It also allows to easily calculate the Point Spread Function, the Optical Transfer Function and the Modulation Transfer Function which are very useful functions quantifying the optical behavior of optical systems [23] including the system of the human eye. Moreover, the scalar theory based compound system simplifies the calculation of optical performance criteria and image quality criteria such as the Optical Transfer Error [24].
In perspectives, the approach of compound systems based on scalar theory of diffraction will be applied to the synthesis of fiber components for use in high bit rate optical telecommunications.

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